

# MODELING OF IMPEDANCE COLLAPSE IN HIGH-VOLTAGE DIODES

P. J. Turchi<sup>1</sup> and R.E. Peterkin, Jr.<sup>2</sup>

Air Force Research Laboratory  
Kirtland AFB, NM, USA

## ABSTRACT

Electron-beam diodes driven by fast-rising, high-voltage pulses often operate with cold cathodes for which the presence of a plasma adjacent to the cathode surface is essential to obtain adequate electron emission. A consequence of such surface plasma, however, is closure of the interelectrode gap by plasma motion. The diode impedance decreases with time, adversely affecting the efficiency of coupling to the power source. Plasma closure of the diode gap also limits the length of the electron beam pulse, and the ability to operate the diode repetitively at high frequency. Resistive heating of the plasma competes with work performed in expanding the plasma and heat transfer to the cold-cathode boundary. The resulting closure speed is calculated, using an MHD code, and found to agree well with results of experiments using organic-cloth cathodes at 35 kV. Computed plasma speeds are typically 8 - 12 km/s, and are relatively insensitive to the applied voltage. Gap closure due to the plasma motion calculated numerically corresponds to estimates based on impedance collapse in the experiments.

## INTRODUCTION

Closure of the interelectrode space in electron-beam diodes operating with cold-cathodes is a well-known phenomena limiting the performance of such devices. Plasma adjacent to the cathode surface is needed to obtain adequate electron emission for space-charge limited flow in the diode. Such plasma, however, can cross the diode gap, resulting in collapse of the diode impedance during the high-voltage pulse. This behavior adversely affects the efficiency of coupling between the electron beam and the power source, limits the duration of the electron-beam pulse, and may preclude operation at desired repetition rates. It is useful, therefore, to understand factors that determine plasma closure in electron-beam diodes in the context of theoretical modeling that can then be employed to examine directions for improvement. The first step in such modeling consists of simulating existing experiments using techniques that can be extended to more complex possibilities.

The simplest representation of plasma closure in a diode comprises a thin layer of uniform plasma between a solid cathode surface and a region representing the vacuum in the diode gap. The physical model consists of expansion of the plasma across the gap, with resistive heating compensating for the loss of internal energy to work and heat transfer. Even this elementary model is too complex for accurate analytical treatment, so numerical techniques are necessary. Estimates of particle density and temperature for the surface plasma suggest that a continuum approach would be appropriate for the motion of the main portion of the plasma. The MACH2 2-1/2 dimensional MHD code is therefore a reasonable choice of calculational tool. Such a code is, of course, clearly inadequate to model the behavior of an electrically non-neutral, non-continuum flow.

The essential feature, however, that must be calculated in order to compute the dynamics and thermodynamics of the surface plasma is the current density. For MACH2, this merely requires that the resistivity of the "vacuum" region has a value corresponding to space-charge limited flow for the instantaneous values of applied voltage and vacuum gap. For most situations, this resistivity is

---

<sup>1</sup> Professor of Aerospace Engineering, on sabbatical leave from The Ohio State University.

<sup>2</sup> Technical Leader, Center for Plasma Theory and Computation.

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>JUN 1997</b>		2. REPORT TYPE <b>N/A</b>		3. DATES COVERED <b>-</b>	
4. TITLE AND SUBTITLE <b>Modeling Of Impedance Collapse In High-Voltage Diodes</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Air Force Research Laboratory Kirtland AFB, NM, USA</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release, distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>See also ADM002371. 2013 IEEE Pulsed Power Conference, Digest of Technical Papers 1976-2013, and Abstracts of the 2013 IEEE International Conference on Plasma Science. Held in San Francisco, CA on 16-21 June 2013. U.S. Government or Federal Purpose Rights License.</b>					
14. ABSTRACT <b>Electron-beam diodes driven by fast-rising, high-voltage pulses often operate with cold cathodes for which the presence of a plasma adjacent to the cathode surface is essential to obtain adequate electron emission. A consequence of such surface plasma, however, is closure of the interelectrode gap by plasma motion. The diode impedance decreases with time, adversely affecting the efficiency of coupling to the power source. Plasma closure of the diode gap also limits the length of the electron beam pulse, and the ability to operate the diode repetitively at high frequency. Resistive heating of the plasma competes with work performed in expanding the plasma and heat transfer to the cold-cathode boundary. The resulting closure speed is calculated, using an MHD code, and found to agree well with results of experiments using organic-cloth cathodes at 35 kV. Computed plasma speeds are typically 8-12 kmh, and are relatively insensitive to the applied voltage. Gap closure due to the plasma motion calculated numerically corresponds to estimates based on impedance collapse in the experiments.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>SAR</b>	18. NUMBER OF PAGES <b>6</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

sufficiently high to permit magnetic-flux diffusion rapidly through the "vacuum" region, which results in nearly uniform current density normal to the surface of the plasma layer. Thermal expansion of the plasma decreases the effective diode gap, but the resistivity of the vacuum region may be continually adjusted to maintain the proper value of space-charge limited current density during the voltage pulse. While this technique cannot simulate the actual behavior of the particle flow in the vacuum, it is adequate for providing the correct power to the plasma in order to compute the motion of the main portion of the plasma mass. Detailed consideration may then be given to penetration of electric field into the plasma in defining the position of the electron emitting surface. This is accomplished in terms of the local values of the Debye length at the vacuum edge of the plasma, aided by the condition of electric field equal to zero for emission into space-charge limited flow.

## USE OF THE MACH2 CODE

The MACH2 code [1] allows numerical solution of the equations for 2-1/2 dimensional, unsteady, MHD flow. It uses an arbitrary Lagrangian-Eulerian (ALE) formulation in combination with a multiple block format that permits application to complex geometries. In addition to classical transport properties and ideal gas formulas, MACH2 operates with the SESAME tables for equation-of-state and transport properties of a variety of materials. Anomalous resistivity coefficients based on plasma/beam microinstabilities are also available, as are options for optically-thin and both equilibrium and non-equilibrium diffusion models for radiative transport. Several opportunities exist as well for coupling plasma dynamics to external electrical sources, including prescribed voltage or current waveforms and self-consistent solution of lumped-element circuitry.

MACH2 is nevertheless a continuum formulation, based on electrical quasi-neutrality and local (two- or three- temperature) thermodynamic equilibrium. Its application to problems in an electron-beam diode is restricted to regions where the plasma density is high enough to satisfy these physical constraints. Plasma created at the cathode surface by explosion of asperities at the cathode surface, or by injection from a continuum plasma source, may satisfy the continuum, quasi-neutral, LTE conditions adequately for the purpose of calculating plasma motion. The very low density, non-neutral region between the emitting surface and the anode certainly does not. MACH2, however, permits the electrical resistivity of a region or material (defined, for example, by its low mass-density) to be assigned quite flexibly. Thus, very low-density material in the vacuum gap can be given, at any instant in the calculation, a value of resistivity that will provide the correct total impedance corresponding to space-charge limited flow. For the applied voltage, therefore, the correct total current will be delivered through the expanding surface-plasma. Implicit in this approach is the approximation that the basically one-dimensional formulation of space-charge limited flow will still be adequate to describe operation of the vacuum region. To correct the value of the current from the Child-Langmuir formula, in order to obtain the actual experimental current before significant impedance collapse has occurred due to plasma motion, a value of perveance is calculated. This correction factor is then held constant during diode closure.

MACH2 has not been used to calculate the actual explosion of surface asperities to create the initial plasma. Instead, the conditions of this initial plasma (mass density, electron and heavy-particle temperatures, and initial thickness) are assumed to be uniform, and treated as parameters in analyzing the subsequent plasma expansion. For properties, such as the closure speed, non-dimensionalization suggests that the the actual value of initial mass-density and initial plasma thickness are not crucial. Similarly, low estimates of the initial temperature might be self-correcting due to enhanced resistive heating. (Over-estimates of the temperature, however, will simply cause the plasma to expand too quickly in comparison to experimental data.)

## SPECIFICATION OF THE ELECTRON-EMISSION SURFACE

Before proceeding to solve the dynamics of plasma closure, and to examine the sensitivities of the model to the choice of initial plasma conditions, it is necessary to develop a specification for the position of the electron-emitting surface. For the simple case of emission from a solid surface, the diode gap is merely the geometric distance between the electrodes, to the level of atomic dimensions. Emission from a plasma surface is less well-defined and can involve uncertainties in effective diode-gap

on the order of the gradient scale-length for the plasma density, the mean free path for electron-electron collisions, the (local) Debye length in the plasma, or some other condition. For the present calculations, we have chosen to define the electron-emitting surface of the plasma based on the condition that the electron velocity-distribution is significantly distorted from a Maxwellian distribution due to the electric field near the emitting surface. Algebraically, this condition is written as:

$$E(x) \lambda_D \geq kT_e \quad (1)$$

where  $\lambda_D$  is the Debye length based on the local plasma temperature  $T_e$  and the local electron density, which varies sharply from the plasma layer to the vacuum region. The local electric field,  $E(x)$ , would be zero exactly at the emitting surface, as a condition of space-charge limited flow, but has a finite value at a distance of a Debye length into diode gap, i.e.,  $x = \lambda_D$ . The condition of Eqn. 1 is thus satisfied, for any plasma-electron temperature, when the electron density falls below some value. While the condition could equally well have been written with a factor (of order unity) multiplying the electron temperature, the resulting variations in diode gap would usually be less than the resolution of the calculational grid employed in the present work. Furthermore, within the approximation of using the one-dimensional, Child-Langmuir formula for space-charged limited flow, we monitor the diode gap according to Eqn. 1 only at one position along the surface (e.g., half-radius). A more laborious approach involving local calculation of the diode gap is deferred until warranted by substantially two-dimensional motion of the plasma surface and until an appropriate substitute for the Child-Langmuir formula can be developed, (which probably will require a particle-in-cell calculation).

## CALCULATIONAL RESULTS

The first application of the present model has been to a set of experiments [2] performed with carbon-cloth cathodes at relatively modest applied voltages ( $\sim 35$  kV). These experiments were chosen for simulation because the clean and simple voltage waveform (linear rise and fall surrounding a relatively long constant value) allowed less uncertainty in comparing theoretical and experimental behavior. The experimental arrangement comprises a cylindrical cathode, 2 cm in radius, with a circular endface covered by the emitting carbon-cloth, separated by a gap of 0.5 cm from a screen anode; (the experiments were directed toward virtual-cathode operation for microwave production.) Figure 1 displays the experimental and theoretical waveforms for voltage and current. As previously noted, a perveance factor is included in the calculation to match space-charge limited flow to the experimental current value before significant diode closure has occurred ( $t < 50$  ns). This factor is held constant during the subsequent plasma motion. The computed current waveform agrees rather well with the experimental curve scanned from the published oscillogram. Experimental behavior from 210 to 300 ns is difficult to discern due to relatively high amplitude, high-frequency oscillations on the trace; the calculations assume an inductance of 100 nh in series with the diode, which may be somewhat too high.

Figure 2 provides the calculated voltage and current waveforms along with the diode gap. In these calculations, which serve as a baseline for parameter variations, the initial electron and ion temperatures in the plasma layer are 1.0 eV and 0.1 eV, respectively, and the initial plasma (mass) density is  $10^{-3}$  kg/m<sup>3</sup>. (These values are chosen arbitrarily, but should be relevant to low temperature plasmas.) The material of the plasma is carbon phenolic, available in the SESAME tables, and was chosen in order to account for the complicated molecular composition of the carbon-cloth cathode, which consists of silk and rayon (vs pure carbon). The effective molecular mass of the phenolic is 9.01 amu. The initial thickness of the surface plasma is 0.5 mm, but rapidly increases to about 1 mm, due to the application of Eqn. 1 to the density distribution computed at early times.

The speed of plasma closure is not constant, but corresponds to an average speed of 8 km/s. This value is consistent with the estimates from the experimentally-observed decrease of diode impedance with time (assuming Child-Langmuir behavior). Such consistency, of course, is directly associated with the good agreement between experiment and theory for the diode current. In addition to the overall behavior of diode impedance -collapse due to plasma motion, the MACH2 simulations

provide detailed descriptions of plasma properties and distributions within the diode. Examples of this information are given in Fig. 3, which displays the shape of the interface between plasma and vacuum, and contours of mass density and electron temperature.

### SENSITIVITIES TO PARAMETER CHOICES

The choices of initial values for the plasma layer may be informed by experience, but are certainly arbitrary in the present work. It is necessary, therefore, to explore the effects of other values on the basic behavior of diode closure. Three important parameters are the initial values of electron temperature, ion temperature and mass density. In the following comparisons, the voltage waveform will be maintained, and only one of these parameters will be varied at a time; the other parameters will be held at their values for the baseline case previously discussed.

Figure 4 displays the current and diode gap histories for initial values of electron temperature that are factors of two lower and higher than the baseline case, 0.5 eV and 2.0 eV, respectively. The lower value results in little change in diode gap and corresponding small change in the diode impedance with time; the current follows the voltage waveform. On the other hand, the higher value of initial electron temperature results in substantial faster diode closure and a much higher value of diode current than is observed experimentally. Comparison of the diode gap histories with that of the baseline case indicates the importance of the early development of closure speed on subsequent decrease of the diode gap and rapid increase of diode current (which scales inversely with the square of the gap). Higher current density helps to maintain the plasma internal-energy as the plasma expands across the gap. The early development of closure speed in the higher temperature case is simply a result of higher (electron) pressure.

In Fig. 5, the initial value of electron temperature is 1.0 eV, but the initial values of ion temperature have been increased by factors of five and ten over the baseline case, 0.5 eV and 1.0 eV, respectively. Higher initial temperatures for the ions also means higher plasma pressure (at fixed mass density), so closure speeds develop faster than in the baseline case. There is not as much sensitivity, however, to variation in ion temperature (vs initial electron temperature) because the ions in the baseline case rapidly warm up to above 0.5 eV due to heat transfer from the electrons. The effect of changing the initial value of mass density is much less pronounced because, at fixed initial values of electron and ion temperatures, the plasma pressure scales approximately with the mass density; (variations are possible due to changes in ionization with density). Thus, the basic expansion speed of the plasma remains approximately the same as for the baseline case. The results of this are observed in Fig. 6 for which factors of ten higher and lower values of initial mass density are used.

To the extent that the simulations have captured the actual behavior of diode closure, the agreement of the baseline case with the experimental data suggests that the properties of the initial plasma are reasonably given by electron and ion temperatures of 1 eV and 0.1 eV, respectively. The initial mass density, however, could be higher or lower by almost a factor of ten without disturbing this agreement very much.

### EFFECTS OF APPLIED VOLTAGE

The present model can be employed to examine the effects of variation of circuit operation on diode closure. For example, if the same temporal waveform is provided to the diode with different amplitudes, the change in diode closure (and thus diode impedance history) can be predicted. Figure 7 displays the calculated diode gap histories as the amplitude of the applied voltage is increased by factors of 1.5 - 3 over the baseline case. The average closure speed only varies by a factor of 50 %, which agrees with experience in a variety of cold-cathode diodes. The peak speed, however, shows greater sensitivity to applied voltage. Examination of the two-dimensional distributions of plasma in the diode indicate that greater nonuniformity, especially faster closure near the centerline of the diode may be responsible for these higher speeds elsewhere in the diode; (recall that the diode gap is only monitored at mid-radius in these calculations). The basic result of the relative insensitivity of diode impedance-

collapse to applied voltage is, perhaps, the only claim that may be made until the present calculations are extended to more two-dimensional treatment of current flow in the vacuum.

## CONCLUDING REMARKS

It appears that MHD codes, such as MACH2, can be successfully employed to study plasma phenomena in high current-density diodes. The basis for this success at present is the application to diode problems for which one-dimensional approximations to the current flow in the vacuum are adequate. Also, by invoking the condition of space-charge limited flow, ( $E(0) \approx 0$ ), the specification of the electron-emitting surface is facilitated. For two- or three-dimensional problems, recourse to particle-in-cell (PIC) techniques to calculate the local current density through the plasma layer will probably be necessary. For some problems, the development of the current flow and charge distribution in the vacuum may occur on a timescale much shorter than the hydrodynamic expansion times for the surface plasma. This situation may then allow calculation of diode behavior with surface plasmas by a rapid iteration between PIC codes, run for several nanoseconds, and hydrocodes running over a microsecond, without encountering the difficulties of including plasma motion within the PIC code itself.

## ACKNOWLEDGEMENTS

This work was performed while one of the authors (PJT) was on sabbatical leave from The Ohio State University, Columbus, OH, and partially supported by the USAF Phillips Laboratory (PL/WS), Kirtland AFB, NM, through the Air Force Office of Scientific Research (AFOSR/NA), Washington, DC.

## REFERENCES

1. R.E. Peterkin, Jr., et al, "MACH2: A Reference Manual", Weapons Laboratory, Kirtland AFB, NM.
2. R.J. Adler, et al, Rev. Sci. Instrum. 56 (5) 1985



